The Hawking evaporation process of rapidly-rotating black holes: An almost continuous cascade of gravitons

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(Dated: June 19, 2015)

It is shown that rapidly-rotating Kerr black holes are characterized by the dimensionless ratio $\tau_{\text{gap}}/\tau_{\text{emission}} = O(1)$, where τ_{gap} is the average time gap between the emission of successive Hawking quanta and τ_{emission} is the characteristic timescale required for an individual Hawking quantum to be emitted from the black hole. This relation implies that the Hawking cascade from rapidly-rotating black holes has an almost continuous character. Our results correct some inaccurate claims that recently appeared in the literature regarding the nature of the Hawking black-hole evaporation process.

I. INTRODUCTION

Working within the framework of semi-classical general relativity, Hawking [1] has revealed that black holes are actually not completely black. In particular, it was shown [1] that black holes are characterized by thermally distributed emission spectra [2]. This intriguing finding is certainly one of the most important theoretical predictions of modern physics.

It has recently been pointed out [3] that the Hawking radiation flux out of spherically-symmetric (non-rotating) Schwarzschild black holes is extremely sparse (see [4–6] for earlier discussions of this black-hole property). In particular, denoting by τ_{gap} the average time gap between the emissions of successive black-hole quanta, and by τ_{emission} the characteristic timescale required for an individual Hawking quantum to be emitted from the black hole, one finds the characteristic dimensionless ratio [3–6]

$$\eta \equiv \frac{\tau_{\text{gap}}}{\tau_{\text{emission}}} = O(10^2) \tag{1}$$

for Schwarzschild black holes.

The relation (1) implies that the quantum radiation flux of Schwarzschild black holes is indeed sparse. Namely, the average time gap between the emissions of successive Hawking quanta out of a Schwarzschild black hole is very large on the timescale $2\pi/\omega$ [see Eq. (8) below] set by the characteristic energy (frequency) of these emitted quanta. Thus, one may safely conclude that the Hawking quanta of an evaporating Schwarzschild black hole are typically emitted from the black hole one at a time [3–6].

It was also claimed in [3] that adding angular momentum to the black hole makes the dimensionless ratio η even *larger*, thus making the Hawking emission spectra of rotating Kerr black holes even more sparse than the corresponding emission spectrum (1) of the non-rotating Schwarzschild black hole. Namely, it was claimed in [3] that

$$\eta_{\text{Kerr}} \geq \eta_{\text{Schwarzschild}} \ . \tag{2}$$

As we shall show in this paper, the claim (2) of [3] is actually erroneous. In particular, explicit calculations to be carried below reveal that, for rapidly-rotating Kerr black holes, $\eta(\bar{a})$ is a *decreasing* function of the dimensionless black-hole angular momentum $\bar{a}$ [7]. Moreover, as we shall show below, near-extremal Kerr black holes are actually characterized by the relation $\eta(\bar{a} \to 1) = O(1)$.

II. THE HAWKING EVAPORATION PROCESS OF RAPIDLY-ROTATING KERR BLACK HOLES

We study the Hawking emission of gravitational quanta by rotating Kerr black holes. The Bekenstein-Hawking temperature of a Kerr black hole and the angular velocity of its horizon are respectively given by the relations [8]

$$T_{\text{BH}} = \frac{\hbar(r_+ - r_-)}{4\pi(r_+^2 + a^2)} \quad \text{and} \quad \Omega_{\text{H}} = \frac{a}{2Mr_+} \ , \tag{3}$$

where $r_{\pm} = M \pm (M^2 - a^2)^{1/2}$ are the (inner and outer) horizon radii of the black hole.

The Hawking emission rate (that is, the number of quanta emitted per unit of time) out of a rotating Kerr black hole is given by the Hawking relation [1, 9]

$$\dot{\mathcal{N}} \equiv \frac{dN}{dt} = \frac{\hbar}{2\pi}\sum_{l,m}\int_0^{\infty} d\omega \frac{\Gamma}{e^x - 1} \ , \tag{4}$$

where $x \equiv \hbar(\omega - m\Omega_{\rm H})/T_{\rm BH}$. Here l and m (with $l \geq |m|$) are respectively the spheroidal harmonic index and the azimuthal harmonic index of the emitted quanta, and $\Gamma = \Gamma_{lm}(\omega;\bar{a})$ are the frequency-dependent gray-body factors [9]. These dimensionless absorption probabilities quantify the imprint of passage of the emitted black-hole quanta through the effective curvature potential which characterizes the black-hole spacetime.

The Hawking emission rate $\dot{\mathcal{N}}(\bar{a})$ [see Eq. (4)] can be computed along the lines of the numerical procedure described in [9]. In particular, one finds that, for rapidly-rotating Kerr black holes, the Hawking emission spectrum is greatly dominated by gravitational quanta with the angular indices [9, 10]

$$l = m = s = 2 \ . \tag{5}$$

Moreover, the characteristic thermal factor that appears in the denominator of (4) implies that, for rapidly-rotating (near-extremal, $T_{\rm BH} \to 0$) black holes, the emission of high energy quanta with $\omega > m\Omega_{\rm H}$ is exponentially suppressed. Thus, for rapidly-rotating Kerr black holes, the Hawking emission spectra are effectively restricted to the regime

$$0 \leq \omega \lesssim m\Omega_{\rm H} + O(T_{\rm BH}/M^2) \ . \tag{6}$$

The reciprocal of the black-hole emission rate,

$$\tau_{\rm gap} = \frac{1}{\dot{\mathcal{N}}} \ , \tag{7}$$

determines the average time gap between the emissions of successive Hawking quanta. On the other hand, the characteristic timescale required for each individual Hawking quantum to be emitted from the black hole, $\tau_{\rm emission}$, can be bounded from below by the time-period it takes to the corresponding emitted wave field to complete a full oscillation cycle. That is [11],

$$\tau_{\rm emission} \geq \tau_{\rm oscillation} = \frac{2\pi}{\bar{\omega}} \ , \tag{8}$$

where $\bar{\omega}$ is the characteristic (average) frequency of the emitted Hawking quanta [12].

In Table I we display the characteristic dimensionless ratio $\tau_{\rm gap}/\tau_{\rm oscillation}$ for rapidly-rotating Kerr black holes [13]. One finds that $\eta(\bar{a})$ is a *decreasing* function of the dimensionless black-hole angular momentum $\bar{a}$. In particular, we find that near-extremal Kerr black holes are characterized by the relation

$$\eta(\bar{a} \to 1) = O(1) \ . \tag{9}$$

$\bar{a} \equiv J/M^2$	0.90	0.96	0.99	0.999	1.0
$\tau_{\rm gap}/\tau_{\rm oscillation}$	13.5	5.5	2.5	1.5	1.2

TABLE I: The characteristic dimensionless ratio $\tau_{\rm gap}/\tau_{\rm oscillation}$ of rapidly-rotating Kerr black holes. Here $\tau_{\rm gap}$ is the average time gap between the emission of successive Hawking quanta [see Eq. (7)] and $\tau_{\rm oscillation}$ is the characteristic oscillation period of the emitted wave field [see Eq. (8)]. One finds that $\eta(\bar{a})$ is a *decreasing* function of the dimensionless black-hole angular momentum $\bar{a}$. In particular, near-extremal Kerr black holes are characterized by the relation $\eta(\bar{a} \to 1) = O(1)$.

III. SUMMARY

It has long been known [3–6] that the Hawking radiation flux out of Schwarzschild black holes is extremely sparse. In particular, these spherically-symmetric black holes are characterized by the large dimensionless ratio $\eta \equiv \tau_{\rm gap}/\tau_{\rm emission} = O(10^2)$ [see Eq. (1)]. As recently pointed out in [3], this relation implies that the individual Hawking quanta emitted from a Schwarzschild black hole are well separated in time.

It was recently claimed in [3] that adding angular momentum to the emitting black hole makes the dimensionless ratio η even larger [see Eq. (2)], thus making the Hawking radiation spectra of rotating Kerr black holes even sparser than the corresponding emission spectrum of the (non-rotating) Schwarzschild black hole.

In this brief report we have explicitly shown that the claim (2) made in [3] is actually erroneous. In particular, explicit calculations reveal that, for rapidly-rotating Kerr black holes, $\eta(\bar{a})$ is actually a *decreasing* function of the dimensionless black-hole angular momentum $\bar{a}$. Moreover, we have shown that near-extremal Kerr black holes are characterized by the relation $\eta(\bar{a} \to 1) = O(1)$.

The relation $\tau_{\text{gap}}/\tau_{\text{emission}} = O(1)$ [see Eq. (9)] implies that the Hawking cascade of gravitons from rapidly rotating Kerr black holes has an almost *continuous* character. Stated in a more picturesque way, we can say that, on average, there is a gravitational quantum leaving the (rapidly-rotating) black hole at any given moment of time.

ACKNOWLEDGMENTS

This research is supported by the Carmel Science Foundation. I thank Yael Oren, Arbel M. Ongo, Ayelet B. Lata, and Alona B. Tea for stimulating discussions.

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[7] Here $\bar{a} \equiv J/M^2$ is the dimensionless angular momentum of the Kerr black hole.
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[10] The last equality in (5) refers to gravitational quanta, which are characterized by the spin parameter $s = 2$.
[11] Note that there is a factor 2π difference between our definition of $\tau_{\text{oscillation}}$ and the corresponding definition used in [3]. We believe that the time period $2\pi/\omega$ required by the emitted wave field to complete a full oscillation cycle [see Eq. (8)] provides a natural lower bound on the characteristic emission timescale τ_{emission} of the corresponding Hawking quantum.
[12] Using in (8) the peak frequency ω_{peak} of the Hawking emission spectrum instead of the average frequency $\bar{\omega}$ of the Hawking emission spectrum would merely change the dimensionless ratio η by a factor of order unity.
[13] It is worth emphasizing again that this ratio provides an upper bound on the actual value of $\eta(\bar{a}) \equiv \tau_{\text{gap}}/\tau_{\text{emission}}$, see Eq. (8).